

2. Given vector field $F(x, y) = (3x^2y^2 + 1)i + 2x^3yj$, then

a) Show that the function F is conservative

b) Find the potential function $f(x, y)$ for $F(x, y)$

c) Evaluate line integral $\int_C F \cdot dr$ from a point $(0, 0)$ to $(1, -2)$

$$a) F(x, y) = (3x^2y^2 + 1)i + 2x^3yj$$

$$\Rightarrow M(x, y) = 3x^2y^2 + 1, \quad N = 2x^3y$$

$$M_y = \underline{6x^2y} = N_x \quad \text{or } \text{curl } F = 0$$

$\therefore F$ is conservative

b) Since F is conservative,

$$F = \nabla f$$

$$\Rightarrow (3x^2y^2 + 1, 2x^3y) = (f_x, f_y)$$

$$\Rightarrow f_x = 3x^2y^2 + 1 \quad \& \quad f_y = 2x^3y$$

$$\Rightarrow f = \int (3x^2y^2 + 1) dx + g(y)$$

$$= x^3y^2 + x + g(y)$$

$$\Rightarrow f_y = 2x^3y + g'(y) = 2x^3y$$

$$\Rightarrow g'(y) = 0$$

$$\Rightarrow g(y) = 0$$

$\therefore f(x, y) = \underline{x^3y^2 + x}$ is the potential fn

$$c) \text{ Line integral } \int_C F \cdot dr = f(1, -2) - f(0, 0)$$

$$= 1^3(-2)^2 + 1 - (0^3 \cdot 0^2 + 0)$$

$$= \underline{\underline{5}}$$